

Centre Number						Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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7	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2010

Mathematics

MM04

Unit Mechanics 4

Thursday 24 June 2010 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

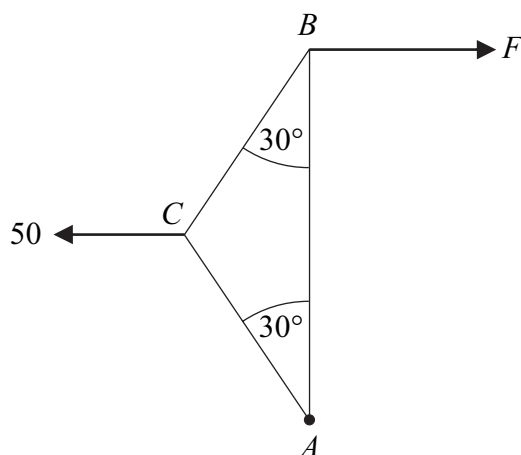
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.



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Answer **all** questions in the spaces provided.

- 1** A framework consists of three light inextensible smoothly jointed rods AB , BC and CA . Rods BC and CA each have length 2 metres and angle $BAC = \text{angle } ABC = 30^\circ$. The framework is freely pivoted to a fixed support at A . Two horizontal forces, of magnitudes 50 newtons and F newtons, act on the framework. The system is in equilibrium in a vertical plane with AB vertical, as shown in the diagram.



- (a) By taking moments about A , find F . (2 marks)
- (b) State the magnitude and direction of the reaction force acting on the framework at A . (2 marks)
- (c) (i) Find the magnitude of the force in the rod BC . (2 marks)
- (ii) Find the magnitude of the force in the rod AB . (2 marks)

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- 2** Stephanie is practising a ballet dancing routine. As part of the routine, she rotates about a vertical axis through her centre of mass.
- (a)** When both her arms are fully extended, her moment of inertia about her axis of rotation is 0.6 kg m^2 and her angular speed is 3 rad s^{-1} . Find her angular momentum. (2 marks)
- (b)** Stephanie now lowers her arms until they are vertical. Her moment of inertia in this position is 0.45 kg m^2 . Find her angular speed when her arms are vertical. (2 marks)

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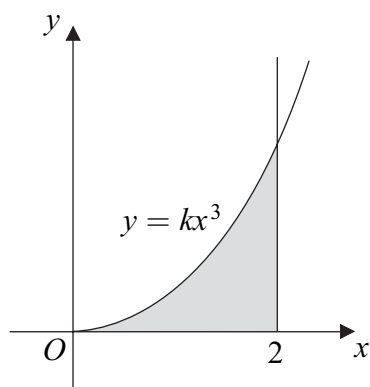
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- 3 A uniform lamina is bounded by the curve $y = kx^3$, the line $x = 2$ and the x -axis, as shown in the diagram.



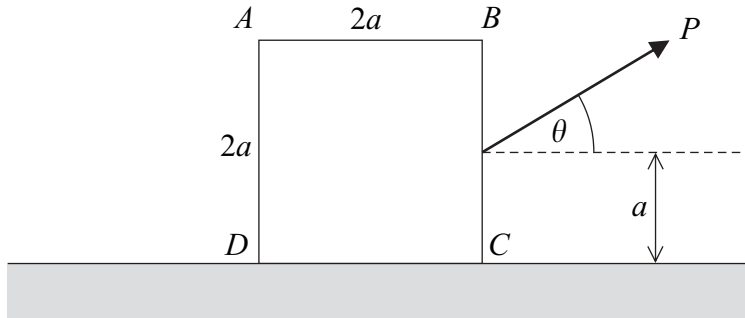
- (a) Find an expression for the area of the lamina in terms of k . (2 marks)
- (b) Find the x -coordinate of the centre of mass of the lamina. (4 marks)
- (c) The y -coordinate of the centre of mass of the lamina is 8.
- (i) Determine the value of k . (4 marks)
- (ii) The lamina is freely suspended from the corner at the origin O . Find the acute angle between the straight edge at the point of suspension and the vertical. (3 marks)

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A uniform cube, of side $2a$ and mass m , rests on a rough horizontal plane. The diagram shows a vertical cross-section $ABCD$ through the centre of mass of the cube.



A force, of magnitude P , is applied at the mid-point of BC . This force acts in the plane $ABCD$ and makes an angle θ with the horizontal. The coefficient of friction between the cube and the plane is μ .

- (a) In the case where the cube does not slide but is on the point of toppling about the edge through C , find an expression for P in terms of m , g and θ . (3 marks)
- (b) In the case where the cube remains upright but is on the point of sliding along the plane, show that $P = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$. (4 marks)
- (c) Find an inequality that μ must satisfy if the cube slides before it topples. (3 marks)
- (d) Would your answer in part (c) change if the mass of the cube were doubled? Explain why. (2 marks)

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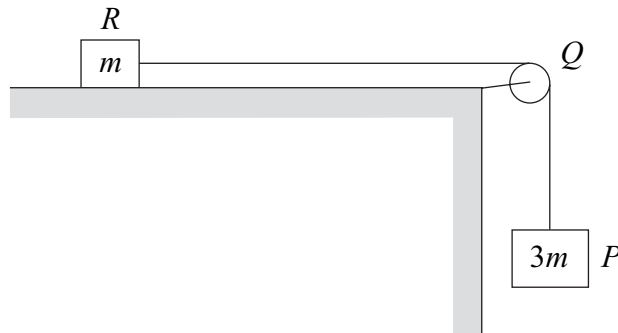
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A pulley Q is fixed to the edge of a smooth horizontal table. The pulley can rotate freely in a vertical plane about a horizontal axis through its centre.

A light inextensible string runs over the pulley, connecting a block R , of mass m , to a block P , of mass $3m$. The block R is held at rest on the table with block P hanging freely, as shown in the diagram.



Model the pulley as a uniform disc of mass $12m$ and radius r . Model the blocks as particles.

- (a) Write down the moment of inertia of the pulley about the horizontal axis through its centre and perpendicular to its plane. (1 mark)
- (b) Block R is released. In the subsequent motion, R moves on the table. The string between P and Q is vertical and has tension T_1 . The string between Q and R is horizontal and has tension T_2 . The pulley has angular acceleration $\ddot{\theta}$. Assume that the string does not slip and that R does not reach the pulley.
- (i) Show that $T_1 - T_2 = 6mr\ddot{\theta}$. (3 marks)
- (ii) Show that $\ddot{\theta} = \frac{3g}{10r}$. (6 marks)
- (iii) Find T_1 and T_2 in terms of m and g . (3 marks)

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6 Two forces, $2\mathbf{i} + a\mathbf{k}$ and $-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, act at the points whose coordinates are $(1, 0, 3)$ and $(-1, 2, 0)$ respectively.

(a) Show that the resultant moment of these forces about the origin is $6\mathbf{i} + (9 - a)\mathbf{j} + 3\mathbf{k}$. (5 marks)

(b) This system is equivalent to a force \mathbf{F} that acts at the origin together with a couple of magnitude 7.

(i) Show that one possible value of a is 7 and find the other possible value of a . (4 marks)

(ii) In the case where $a = 7$, find \mathbf{F} . (2 marks)

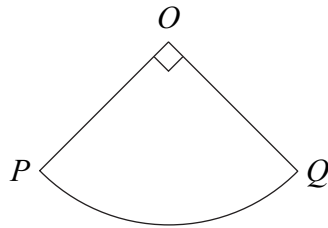
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7 (a) Prove by integration that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis through one end of the rod and perpendicular to the rod is $\frac{4}{3}ma^2$. (4 marks)

(b) The diagram shows a simple model of a theme park swingboat ride.



The model consists of two uniform rods, OP and OQ , and a seat in the form of a circular arc PQ with centre O . Each rod has mass m and length $2a$. The seat is of mass $4m$ and angle $POQ = 90^\circ$. The rods and the seat are rigidly fixed together and the model is free to rotate about a horizontal axis through O . The axis is perpendicular to the plane of OPQ .

(i) Show that the moment of inertia of the model about this axis is $\frac{56ma^2}{3}$. (4 marks)

(ii) The centre of mass of the model is at a distance of approximately $1.44a$ from the point O . The model is rotated until OQ is horizontal, with P vertically below O , and is then released from rest.

In the case where $a = 1.5$, find the greatest angular speed during the subsequent motion. (6 marks)

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END OF QUESTIONS

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